

PAPER-1 (B.E. / B.TECH)

QUESTIONS & SOLUTIONS

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16 March, 2021

03:00 pm to 06:00 pm

SHIFT-2



Duration : 3 Hours

Max. Marks : 300

SUBJECT - MATHEMATICS

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MATHEMATICS

1. Number of solution of the equations $\sin^{-1} \frac{3x}{5} + \sin^{-1} \frac{4x}{5} = \sin^{-1} x$; $x \in [-1, 1]$ is

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (4)

Sol. Taking sine both sides

$$\begin{aligned} \frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} &= x \\ \Rightarrow 3x \sqrt{25 - 16x^2} &= 25x - 4x \sqrt{25 - 9x^2} \\ \Rightarrow x = 0 \text{ or } 3\sqrt{25 - 16x^2} &= 25 - 4\sqrt{25 - 9x^2} \\ \Rightarrow 9(25 - 16x^2) &= 625 - 200\sqrt{25 - 9x^2} + 16(25 - 9x^2) \\ \Rightarrow 200\sqrt{25 - 9x^2} &= 800 \\ \Rightarrow \sqrt{25 - 9x^2} &= 4 \\ \Rightarrow x^2 &= 1 \\ \Rightarrow x &= \pm 1 \\ \therefore \text{Total number of solution} &= 3 \end{aligned}$$

2.

Number of elements	Group-1	Group-2
Number of elements	10	n
Mean	2	3
variance	2	1

If combined variance of both groups is $\frac{17}{9}$, then find 'n'

Ans. (5)

Sol. For group-1 : $\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$

$$\frac{\sum x_i^2}{10} - (2)^2 = 2 \Rightarrow \sum x_i^2 = 60$$

for group-2 : $\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$

$$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$$

Now, combined variance

$$\sigma^2 = \frac{\sum (x_i^2 + y_i^2)}{10 + n} - \left(\frac{\sum (x_i + y_i)}{10 + n} \right)^2$$

$$\Rightarrow \frac{17}{9} = \frac{60 + 10n}{10 + n} - \frac{(20 + 3n)^2}{(10 + n)^2}$$

$$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0 \Rightarrow n = 5$$

3. If $f(x) = \begin{cases} \frac{\cos^{-1}(1-\{x\}^2) \sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

then find α if $f(x)$ is continuous at $x = 0$ (Here $\{x\}$ denotes fractional part of x)

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{\sqrt{2}}$ (3) π (4) no value of α

Ans. (4)

Sol. $RHL = \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1-x^2) \sin^{-1}(1-x)}{x(1-x^2)} = \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1-x^2)}{x}$
 $= \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{-1}{\sqrt{1-(1-x^2)^2}} (-2x)$ (L'Hospital Rule)

$= \pi \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2x^2 - x^4}} = \pi \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2-x^2}} = \frac{\pi}{\sqrt{2}}$

$LHL = \lim_{x \rightarrow 0^-} \frac{\cos^{-1}(1-(1+x)^2) \sin^{-1}(-x)}{(1+x)-(1+x)^3} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{(1+x)[(1+x)^2-1]} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{x^2+2x}$
 $= \frac{\pi}{2} \left(\frac{1}{2} \right) = \frac{\pi}{4}$

As $LHL \neq RHL$ so $f(x)$ is not continuous at $x = 0$

4. Let the circle $x^2 + y^2 - 9x - 2ay + c = 0$ has x -intercept and y -intercept respectively $2\sqrt{2}$ and $2\sqrt{5}$. Find distance of farthest tangent from origin which is perpendicular to $2y + x = 0$

- (1) $\sqrt{6}$ (2) $\sqrt{30}$ (3) $\sqrt{10}$ (4) $2\sqrt{6}$

Ans. (1)

Sol. $2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$, $2\sqrt{a^2 - c} = 2\sqrt{5}$

$\frac{a^2}{4} - c = 2$ $a^2 - c = 5$

$c = \frac{a^2}{4} - 2$

$a^2 = 4c + 8$

$3c + 8 = 5$

$c = -1, a^2 = 4$

Equation of tangent

$y = 2x + c'$

$D = \left| \frac{c'}{\sqrt{5}} \right| = r$

$\frac{c}{\sqrt{5}} = \sqrt{\frac{a^2}{4} + a^2 - c}$

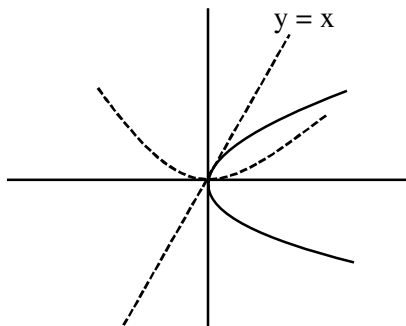
$\frac{c}{\sqrt{5}} = \sqrt{1+5} = \sqrt{30}$

$D = \sqrt{6}$

5. Let image of $y^2 = 4x$ in $y = x$ is a parabola. The equation of tangent from (2, 1) to new parabola is

- (1) $y = x + 1$ (2) $x = y + 1$ (3) $y = 2x - 3$ (4) $y = \frac{x}{2} + 1$

Ans. (2)



Sol.

Image of $y^2 = 4x$ w.r.t. $y = x$ is $x^2 = 4y$

tangent from (2, 1)

$$xx_1 = 2(y + y_1)$$

$$2x = 2(y + 1)$$

$$x = y + 1$$

6. If point of intersection of the curves $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = 4b$, where $b > 4$, lies on the curve

$y^2 = 3x^2$ then find 'b'

- (1) 4 (2) 12 (3) 10 (4) 6

Ans. (2)

Sol. $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ (1)

$$x^2 + y^2 = 4b$$
(2)

$$y^2 = 3x^2$$
(3)

From eq (2) and (3) $x^2 = b$ and $y^2 = 3b$

from equation (1) $\frac{b}{16} + \frac{3b}{b^2} = 1$

$$\Rightarrow b^2 + 48 = 16b$$

$$\Rightarrow b = 12$$

7. Find the area bounded between the curve whose differential equation are given by

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \text{ and } \frac{dx}{dy} = \frac{x^2 - y^2}{2xy}$$

Given that both the curve passes through the point (1, 1)

- (1) $(\pi + 1)$ (2) $(\pi - 1)$ (3) $\left(\frac{\pi}{2} - 1\right)$ (4) $\left(\frac{\pi}{2} + 1\right)$

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx$

$$v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} = -\frac{(v^2 + 1)}{2v}$$

$$\Rightarrow \frac{2v}{v^2 + 1} dv = -\frac{dx}{x}$$

$$\ln(v^2 + 1) = -\ln x + \ln c \Rightarrow v^2 + 1 = \frac{c}{x}$$

$$\Rightarrow \frac{y^2}{x^2} + 1 = \frac{c}{x} \Rightarrow x^2 + y^2 = cx$$

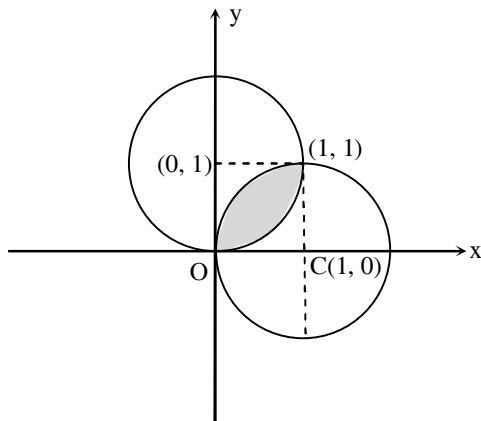
If pass through (1, 1)

$$\therefore x^2 + y^2 - 2x = 0$$

similarly for second differential equation $\frac{dx}{dy} = \frac{x^2 - y^2}{2xy}$

equation of curve is $x^2 + y^2 - 2y = 0$

Now required area is



$$= \left(\frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1 \right) \times 2$$

$$= \left(\frac{\pi}{2} - 1 \right) \text{ sq. units}$$

8. If $\exp \left[\left(\frac{(|z|+3)(|z|-1)}{|z|+1} \right) \log_e 2 \right] \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|$, then find the minimum value of $|z|$

(1) 3

(2) 2

(3) 5

(4) 6

Ans. (1)

Sol. $2^{\frac{(|z|+3)(|z|-1)}{|z|+1}} \geq 2^3 \Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$

$$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$$

$$\Rightarrow |z|^2 - |z| - 6 \geq 0$$

$$(|z| - 3)(|z| + 2) \geq 0$$

$$|z|_{\min} = 3$$

9. Let $f(x) ; (0, \infty) \rightarrow \mathbb{R}$ is defined $f(x+1) = xf(x)$ & $g(x) ; \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$g(x) = \log(f(x))$, then the value of $|g''(5) - g''(1)|$ is

(1) $\frac{205}{144}$

(2) $\frac{144}{205}$

(3) $\frac{144}{113}$

(4) none of these

Ans. (1)

Sol. $g(x+1) = \ln(f(x+1)) = \ln(x f(x)) = \ln x + \ln(f(x))$

$$\Rightarrow g(x+1) - g(x) = \ln x$$

$$g'(x+1) - g'(x) = \frac{1}{x}$$

$$\Rightarrow g''(x+1) - g''(x) = -\frac{1}{x^2} \quad \dots(i)$$

putting $x = 1, 2, 3, 4$ in (i), we get

$$\Rightarrow g''(2) - g''(1) = -1$$

$$g''(3) - g''(2) = -\frac{1}{4}$$

$$g''(4) - g''(3) = -\frac{1}{9}$$

$$g''(5) - g''(4) = -\frac{1}{16}$$

Adding,

$$g''(5) - g''(1) = -1 - \frac{1}{4} - \frac{1}{9} - \frac{1}{16} = -\frac{205}{144}$$

$$\therefore |g''(5) - g''(1)| = \frac{205}{144}$$

10. If $y = y(x)$ the solution of the differential equation $\frac{dy}{dx} + y \tan x = \sin x$ and $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to

- (1) $\frac{1}{2} \ln 2$ (2) $\frac{1}{2\sqrt{2}} \ln 2$ (3) $\frac{1}{\sqrt{2}} \ln 2$ (4) $\ln 2$

Ans. (2)

Sol. Integrating factor = $e^{\int \tan x \, dx} = \sec x$

$\therefore$ solution of the equation is

$$y \sec x = \int \sin x \times \sec x \, dx$$

$$\Rightarrow \frac{y}{\cos x} = \ln(\sec x) + c$$

$$\text{put } x = 0, c = 0$$

$$\therefore y = \cos x \ln(\sec x)$$

$$\text{put } x = \frac{\pi}{4}$$

$$y = \frac{1}{\sqrt{2}} \ln \sqrt{2} = \frac{1}{2\sqrt{2}} \ln 2$$

11. If $f(x) = \begin{vmatrix} 1 + \cos^2 x & \sin^2 x & \cos 2x \\ \cos^2 x & 1 + \sin^2 x & \cos 2x \\ \cos^2 x & \sin^2 x & \sin 2x \end{vmatrix}$, then maximum value of $f(x)$

- (1) $\sqrt{5}$ (2) 5 (3) $\sqrt{3}$ (4) $2\sqrt{5}$

Ans. (1)

Sol. $C_1 \rightarrow C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 2 & 1 + \sin^2 x & \cos 2x \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 0 & 1 & 0 \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = 2 \sin 2x - \cos 2x$$

$$f(x)_{\max} = \sqrt{5}$$

12. The value of integral $\int_0^{10} \frac{[x] \cdot e^{[x]}}{e^{x-1}} dx$ is equal to

- (1) $45(1 - e)$ (2) $55(e - 1)$ (3) $45(e - 1)$ (4) $45(e + 1)$

Ans. (3)

Sol.
$$I = \int_0^{10} [x] \cdot e^{[x]+1-x} dx$$

$$= \int_1^2 e^{2-x} dx + \int_2^3 2e^{3-x} dx + \int_3^4 3e^{4-x} dx + \dots + \int_9^{10} 9e^{10-x} dx$$

$$= -\{(1-e) + 2(1-e) + 3(1-e) + \dots + 9(1-e)\}$$

$$= 45(e-1)$$

13. If $S = \{0,1,2,\dots,6\}$ and a six digit number is formed from it, find the probability it is divisible by '3'.

- (1) $\frac{4}{5}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. $6 \times 6 \times 5 \times 4 \times 3 \times 2$

$$n(5) = 6 \times 6!$$

$$0,1,2,3,4,5,6$$

$$n(\in) = \frac{6! + 2 \times 5 \times 5!}{6 \times 6!}$$

$$= \frac{6+10}{36}$$

$$= \frac{4}{9}$$

14. If two matrices A and B are $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ such that $A = XB$, where $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix}$,

then find k such that $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(1+k^2)b_2^2 \neq 2b_1b_2$.

Ans. 1

Sol. $\therefore A = XB$

$$\therefore \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\Rightarrow a_1 = \frac{1}{\sqrt{3}}(b_1 - b_2) \text{ and } a_2 = \frac{1}{\sqrt{3}}(kb_1 + b_2)$$

Given that

$$a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\Rightarrow \frac{1}{3}(b_1 - b_2)^2 + \frac{1}{3}(kb_1 + b_2)^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\Rightarrow (1+k^2)b_1^2 + 2b_2^2 + 2(k-1)b_1b_2 = 2b_1^2 + 2b_2^2$$

$$\Rightarrow (1+k^2)b_1^2 + 2(k-1)b_1b_2 = 2b_1^2$$

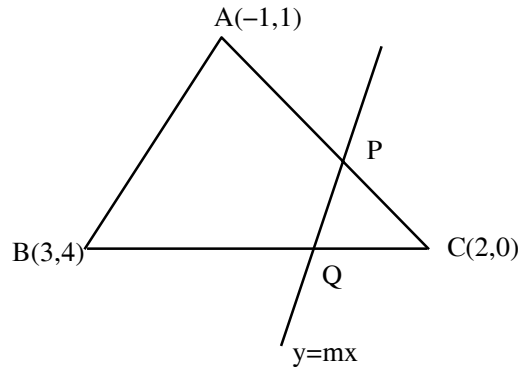
this is only possible when $k = 1$

15. Consider 3 points A(-1, 1), B(3, 4) and C(2, 0). The line $y = mx$ cuts line AC and BC at points P and Q respectively. If area of $\triangle ABC = A_1$, and area of $\triangle PQC = A_2$ and $A_1 = 3A_2$, then positive value of m is

- (1) 1 (2) $\frac{4}{15}$ (3) 2 (4) $\frac{15}{4}$

Ans. (1)

Sol.



$$A_1 = \triangle ABC = \frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix}$$

$$A_1 = \frac{13}{2}$$

Equation of line AC is $y - 1 = \frac{1}{3} (x + 1)$

solve it with line $y = mx$, we get $P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$

Equation of line BC is $y - 0 = 4 (x - 2)$

Solve it with line $y = mx$, we get $Q\left(\frac{-8}{m-1}, \frac{-8m}{m-4}\right)$

$$A_2 = \text{Area of } \triangle PQC = \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-1} & \frac{-8m}{m-4} & 1 \end{vmatrix} = \frac{A_1}{3} = \frac{13}{6}$$

$$\frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ 0 & \frac{2}{3m+1} & 1 \\ 0 & \frac{-8}{m-4} & 1 \end{vmatrix} = \frac{13}{6}$$

$$\frac{1}{2} \left| m(2) \left(\frac{2}{3m+1} + \frac{8}{m-4} \right) \right| = \frac{13}{6}$$

$$2m \left| \left(\frac{m-4+12m+4}{(3m+1)(m-4)} \right) \right| = \pm \frac{13}{6}$$

$$\frac{26m^2}{(3m+1)(m-4)} = \pm \frac{13}{6}$$

16. If $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \dots$. Also, $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$. then find a.

Ans. (16)

Sol. $S_n(x) = (\log_a x) \underbrace{2+3+6+11+\dots}_7$

$$t_n = an^2 + bn + c$$

$$a + b + c = 2 \quad \dots (i)$$

$$4a + 2b + c = 3 \quad \dots (ii)$$

$$9a + 3b + c = 6 \quad \dots (iii)$$

$$\Rightarrow a = 1; b = -2; c = 3$$

$$T_n = n^2 - 2n + 3 = (n-1)^2 + 2$$

$$\therefore S_{24}(x) = 1093 = \log_a x \cdot (4372)$$

$$\Rightarrow x = a^{1/4} \quad \dots (i)$$

$$\text{Also, } S_{12}(2x) = 265 = (\log_a 2x) \cdot (530)$$

$$\Rightarrow 2x = a^{1/2} \quad \dots (ii)$$

from (i) and (2)

$$a^{1/2} = 2a^{1/4}$$

$$\Rightarrow a^2 = 16a \Rightarrow a = 16$$

17. If $\frac{1}{16}, a, b$ are in GP and $\frac{1}{a}, \frac{1}{b}, 6$ are in AP, then

$$72(a+b) =$$

Ans. 14

Sol. $a^2 = \frac{b}{16}$ and $\frac{2}{b} = \frac{1}{a} + 6$

$$\text{Solving, we get } a = \frac{1}{12} \text{ or } a = \frac{-1}{4} [\text{rejected}]$$

$$\text{If } a = \frac{1}{12} \Rightarrow b = \frac{1}{9}$$

$$\therefore 72(a+b) = 72 \left(\frac{1}{12} + \frac{1}{9} \right) = 14$$

18. The number of points lying in the interior of line segments AB, BC, CD, DA of rectangle ABCD are 5, 6, 7 and 9. If α is the number of triangle made from the points by selecting each vertex from different line segments and β is the number of such quadrilaterals then the value of $|\alpha - \beta|$ is

(1) 711 (2) 717 (3) 919 (4) 926

Ans. (2)

Sol. $\alpha = {}^6C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^7C_1 = 378 + 315 + 270 + 210 = 1173$

$$\beta = {}^5C_1 {}^6C_1 {}^7C_1 {}^9C_1 = 1890$$

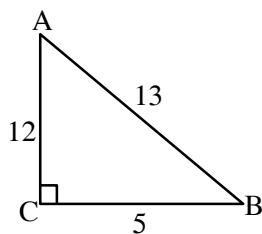
$$\Rightarrow \beta - \alpha = 1890 - 1173 = 717$$

19. Let a, b are sides of triangle where a = 5, b = 12 and area of triangle is 30, then value of $2R + r$

(1) 13 (2) 15 (3) 20 (4) 17

Ans. (2)

Sol. $\angle C = 90^\circ$



$$R = \frac{13}{2}, S = \frac{12+5+13}{2} = 15$$

$$r = \frac{\Delta}{S} = \frac{30}{15} = 2$$

$$2R + r = 13 + 2 = 15$$

20. Find the interval in which $f(x) = \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}$ is increasing

(1) $(-\infty, -1) \cup [0, 1) \cup (1, \infty)$ (2) $(-1, -0) \cup [0, 1) \cup (1, \infty)$
(3) $(-1, -\infty)$ (4) $(-\infty, -1) \cup (-1, 1)$

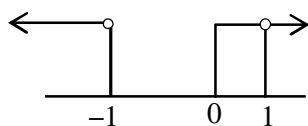
Ans. (1)

Sol. $f'(x) = \frac{x+1}{x-1} \times \frac{x+1-(x-1)}{(x+1)^2} + \frac{2}{(x-1)^2} = \frac{2}{(x-1)(x+1)} + \frac{2}{(x-1)^2}$

$$= \frac{2}{(x-1)} \left(\frac{1}{x+1} + \frac{1}{x-1} \right) = \frac{2}{(x-1)^2(x+1)} \quad (2x)$$

$$= \frac{4x}{(x-1)^2(x+1)}$$

for increasing, $y'(x) \geq 0$



$$\Rightarrow \frac{x}{(x-1)^2(x+1)} \geq 0$$

$$\Rightarrow x \in (-\infty, -1) \cup [0, 1) \cup (1, \infty)$$

- 21.** Consider the set $A = \{2, 3, 4, \dots, 30\}$. A relation R defined on $A \times A$ is an equivalence relation such that $(a, b) R (c, d) \Rightarrow ad = bc$. If $(c, d) = (3, 4)$, then find the number of ordered pair of (a, b)

Ans. (7)

Sol. $(a, b) R (c, d) \Rightarrow ad = bc$

$$\therefore (a, b) R (3, 4) \Rightarrow 4a = 3b \Rightarrow a = \frac{3}{4}b$$

$\Rightarrow b$ is a multiple of 4

$$\therefore (a, b) = (3, 4), (6, 8), (9, 12), (12, 16), (15, 20), (18, 24), (21, 28)$$

i.e., 7 ordered pairs

- 22.** If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and a vector $\vec{c}$ perpendicular to both $\vec{a}$ and $\vec{b}$ and

$$\vec{c} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8 \text{ then the value of } \vec{c} \cdot (\vec{a} \times \vec{b})$$

(1) 90

(2) -88

(3) 80

(4) 78

Ans. (2)

Sol. $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{c} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \lambda [i(1-4) - j(1-2) + k(2-1)]$$

$$\vec{c} = \lambda(-3\hat{i} + \hat{j} + \hat{k})$$

$$\vec{c} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$$

$$\lambda(-3\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$$

$$\lambda(-3-1+3) = 8$$

$$-\lambda = 8$$

$$\lambda = -8$$

$$\vec{c} = -8(\vec{a} \times \vec{b})$$

$$\vec{c} \cdot (\vec{a} \times \vec{b}) = -8|\vec{a} \times \vec{b}|^2 = -8(\sqrt{9+1+1})^2 = -88$$

23. Point (1, -1, 2) is the foot of perpendicular drawn from point (0, 3, 1) on the line

$$\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}. \text{ find the shortest distance between this line and the line}$$

$$\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}.$$

(1) $\frac{61}{\sqrt{1314}}$

(2) $\frac{71}{\sqrt{1314}}$

(3) $\frac{91}{\sqrt{1314}}$

(4) $\frac{31}{\sqrt{1314}}$

Ans. (1)

Sol. Let A(0, 3, 1) and B(1, -1, 2)

d.r's of AB are 1, -4, 1

$$\ell(1) + 3(-4) + 4(1) = 0 \Rightarrow \ell = 8$$

$$\text{B line on } L_1 \Rightarrow \frac{1-a}{8} = -1 = \frac{2-b}{4}$$

$$\Rightarrow a = 9, b = 6$$

$$\Rightarrow L_1 \text{ is } \frac{x-9}{8} = \frac{y-2}{3} = \frac{z-6}{4}$$

$$L_2 \text{ is } \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$$

$$\begin{array}{ccc} 8 & 0 & 3 \\ 8 & 3 & 4 \\ 3 & 4 & 5 \end{array}$$

$$\text{Shortest distance} = \frac{\left| \begin{vmatrix} 8 & 0 & 3 \\ 8 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \right|}{\left| (8\hat{i} + 3\hat{j} + 4\hat{k}) \times (3\hat{i} + 4\hat{j} + 5\hat{k}) \right|}$$

$$= \frac{61}{\left| -\hat{i} - 28\hat{j} + 23\hat{k} \right|} = \frac{61}{\sqrt{1 + 784 + 529}} = \frac{61}{\sqrt{1314}}$$